PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

On the Effect of Temperature on the Specific Inductive Capacity of a Dielectric

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PHILOSOPHICAL TRANSACTIONS.

I. *On the Effect of Temperature on the Specific Inductive Capacity of a Dielectric.*

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THE object of the experiments described in this paper was to learn how the specific inductive capacity of a dielectric is affected by change of temperature.

CAVENDISH observed an increase in the capacity of a glass condenser when it was heated, but gave no measure of the effect. Dr. HOPKINSON observed in light flint glass an increase of $2\frac{1}{2}$ per cent. in the capacity between 12° and 83° C. And Messrs. GIBSON and BARCLAY showed that there is no appreciable change in the case of paraffin between temperatures −12° and 24° C. Except these, no measurements of the effect appear to have been hitherto published.

The present investigation shows an increase of specific inductive capacity with rise of temperature in all the solids* examined, and a decrease in all the liquids except one.

As paraffin, which is a substance comparatively near its melting point at ordinary temperatures, shows no change, these results seem to indicate, as far as they go, that the specific inductive capacity of a substance has a maximum value about the melting point. But it may be questioned whether the data are sufficient as yet to warrant so general an induction.

The relation which CLERK MAXWELL'S electromagnetic theory of light indicates between specific inductive capacity and refractive index makes it interesting to compare the effects of temperature on these two quantities. Four of the liquids

* The results given here for mica, ebonite, and the first specimen of glass are all less than those mentioned by Professor J. J. THOMSON, in his treatise on "Applications of Dynamics to Physics and Chemistry"; because the results quoted there were obtained from an earlier set of observations, in which the precautions for insulation were inferior to those described here.

investigated here are amongst those for which DALE and GLADSTONE have observed the refractive indices at several temperatures. For one of these four the temperature effects show no similarity whatever, for another the relation is fairly close to that indicated by the theory, and for the other two, though not agreeing exactly with the theory, the relation does not differ from it very greatly, not more, perhaps, than might be explained by differences in the specimens used.

The experiments were done in the Cavendish Laboratory, Cambridge, and I am glad of this opportunity of thanking Professor J. J. THOMSON, F.R.S., at whose suggestion they were undertaken, for placing the resources of the Laboratory at my disposal, and for valuable advice during the course of the work.

I. SOLIDS.

The method adopted in the case of solids was to make a condenser with the substance to be experimented upon and observe its capacity at different temperatures. The condenser consisted of a pile of alternate thin sheets of the dielectric and discs of lead foil, with flat plates of iron above and below, and a weight on the top to keep them together. The condenser could not be fixed together in any more permanent way, because the unequal expansion with heat of the different materials would have altered the degree of compression of the pile, and so produced a change of capacity greater than the effect under investigation.

Special precautions were required to secure that the insulation between the two sides of the condenser be as far as possible independent of the temperature. To secure this the condenser was suspended in a stirrup from a bracket about four feet above the air bath in which it was heated. The suspending wires, which formed the connection for the side of the condenser in contact with the stirrup, passed without touching through holes in the top of the air bath, and over a short glass tube varnished with shellac, which was fixed across the bracket above. The connection for the other side of the condenser consisted of a stiff wire, which also passed without touching through a hole in the top of the air bath, and was carried through the glass tube on the bracket. Thus the condenser had no contact with anything about the air bath, and was supported solely by the glass tube on the bracket, which was always cold and perfectly insulating. The perfect freedom of the condenser and its supports was easily tested before each observation by tapping the air bath and seeing that no motion was produced in the suspending wires.

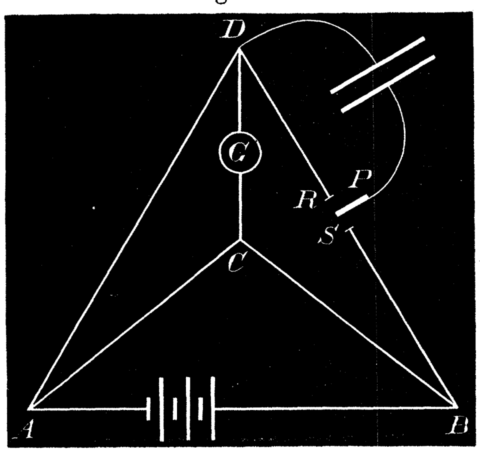
The condenser was heated by raising the bath quickly to a temperature considerably above that aimed at, and then leaving under it a small flame adjusted by experience to give the right temperature. The temperature was read by a thermometer passing through a cork in the top of the bath. About three hours was the shortest time required to ensure that the condenser was uniformly heated throughout; it was usually left a good deal longer.

The condensers needed several heatings before settling down to a steady capacity in the cold state. The temperatures in these preliminary heatings were always above the highest used in the actual observations.

The capacities were measured by the method used by Professor J. J. THOMSON in his determination of the ratio of the electrostatic and electromagnetic units ('Phil. Trans.,' 1883). It is thus described in his paper :—

"In a Wheatstone's bridge, A B C D, with the galvanometer at G, and the battery between A and B, the circuit B D is not closed, but the points B and D are connected with the two poles, R and S, of a commutator, between which a travelling piece, P, moves backwards and forwards; P is connected with one plate of a condenser, the other plate of which is connected with D. Thus when P is in contact with S, the condenser will be charged, and until it is fully charged, electricity will flow into it from the battery; this will produce a momentary current through the various arms of the bridge. When the moving piece P is in contact with R, the two plates of the condenser are connected, and the condenser will discharge itself through D R, and as the resistance of D R is infinitesimal in comparison with the resistance of

Fig. 1.



any other circuit, the discharge of the condenser will not send an appreciable amount of electricity through the galvanometer. Thus, if we make the moving piece P oscillate quickly from R to S, there will, owing to the flow of electricity to the condenser, be a succession of momentary currents through the galvanometer. The resistances are so adjusted that the deflection of the galvanometer produced by these momentary currents is balanced by the deflection due to the steady current through the galvanometer, and the resultant deflection is zero. When this is the case there is a relation between the capacity of the condenser, the number of times the condenser is charged and discharged per second, and the resistances in the various arms of the bridge."

This relation, which is worked out in the paper, is expressed by taking a, b, c, d, g to represent the resistances of A C, A B, A D, B C, D C respectively. The resistances of D R and S B are so small as to be negligible. Then if the condenser has a capacity C, and is charged and discharged n times per second

$$nC = \frac{a\left\{1 - \frac{a^2}{(a+c+g)(a+b+d)}\right\}}{cd\left\{1 + \frac{ab}{c(a+b+d)}\right\}\left\{1 + \frac{ag}{d(a+c+g)}\right\}}.$$

The resistances used were about

$a = 10$,
$b = 8$,
c between 3000 and 6000,
d ,, 6000 ,, 2000,
$g = 4060$ ohms.

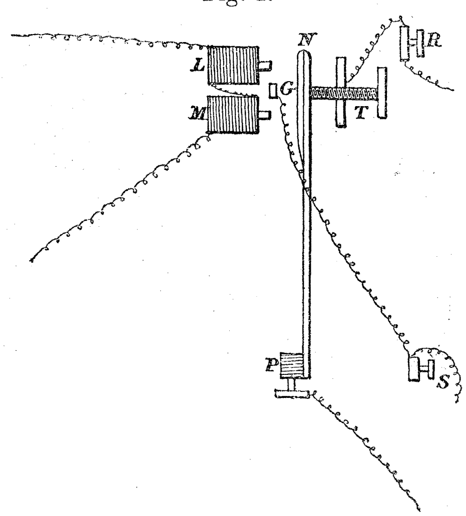
With these values

$$nC = \frac{a}{cd}$$

is correct to within ·2 per cent., and is the formula used here.

The commutator P R S (fig. 2) was that used by Professor THOMSON and is thus described in his paper :—

Fig. 2.



" The current from two GROVE's cells passes first through a tuning-fork interruptor, and then through the coils L M of an electromagnet. P N is a strip of brass with a piece of iron attached to it. When there is no current passing through the electromagnet, the elasticity of the rod P N makes it press against a screw T, which is electrically connected with a binding screw R: when the current passes through the electromagnet, the magnet attracts the iron attached to the rod P N and brings it into contact with the stop G, which is electrically connected with the binding screw S. The letters P, R, S indicate the same points in this figure as in fig. 1. All the places where contact is made by the vibrating piece P N are covered with platinum, and the whole arrangement is fastened down to an ebonite board. As the current passes intermittently through the coils L M of the electromagnet, the vibrating piece strikes alternately against the parts G and T; when it strikes against G the opposite plates of the condenser are connected with the two poles of the battery; when it strikes against T the condenser is discharged (see fig. 1)."

To be able to allow for conduction or absorption in the dielectric by means of observations with forks of different rates, we must know the comparative times during which the vibrating piece P is in contact with G, in contact with T, and travelling from one to the other. I therefore fixed a stiff arm to the end of P and took tracings with it on smoked paper, moved across while the interruptor was vibrating. These tracings showed that when the interruptor is going steadily (this is easily known by the sound) the vibrator is in contact with G and T for equal times. and the time occupied in passing from one to the other is negligible compared to the time of contact.

The battery used was six LECLANCHÉ cells, and the variable resistances were taken from a box by ELLIOTT. Tuning forks were used, making 99, 64, and 49 complete vibrations per second. The forks were worked by a current from the storage cells in the Laboratory.

The coefficients of expansion with temperature of lead, ebonite, and glass are ·000029, ·000077, and ·000009 respectively. That of mica does not seem to have been determined; but it is probably less than that of lead. So that the combined effects of expansion with rise of temperature do not affect the capacity before the fifth decimal place; and, as the results do not profess to be correct to more than four decimal places, the expansion of the materials may be neglected.

Some observations were also taken with ebonite by an electrostatic method based upon that described in MAXWELL'S "Electricity and Magnetism," § 229 (second edition); the only difference being that there was no guard ring. The condenser was compared with a variable condenser by dividing a charge between them, separating them, and connecting opposite poles together and to the electrometer; then, if they were unequal, the difference deflected the electrometer. For this method a special key was required to make the connections quickly and to keep the electrometer to earth until everything except the charge to be measured had been discharged. If the electrometer were not kept to earth it would be deflected in spite of ordinary screens by induction inside the key on account of the high potentials required.

When neither of the condensers being compared has any absorption, there is no difficulty with this method, and any ordinary quadrant electrometer may be used. But when this is not the case the residual discharge will begin to come out as soon as the condensers are connected, and this makes the needle always go ultimately to the same side. The only way to get a balance is to make the variable condenser too large, and gradually diminish it until the initial motion of the needle due to it disappears. This requires a needle with a short time of swing, because with a slow needle the residual charge comes out and overpowers the first effect before the needle has time to show it. With a small ebonite condenser it was just possible to make an ELLIOTT'S electrometer sufficient by making the connection as short as possible; but with a large condenser it was useless. Consequently, this experiment would require an electrometer with a needle of very short time of swing. One with small cylindrical needle and quadrants seems most suitable.

Mica.

The mica condenser was made of twenty-two sheets of brown Muscovite with equilateral triangular markings. The sheets were about $3\frac{1}{2}$ inches diameter, and the thicknesses varied from about a quarter to about a tenth of a millimetre. Some difficulty was found in getting rid of surface conduction on the mica. No cleaning would make it insulate, but in the end perfect insulation was secured by putting a border of shellac varnish round the edges of the sheets. The shellac did not come between the plates of the condenser, and so did not affect the capacity. The highest temperatures in the preliminary heating were less than 140° C.

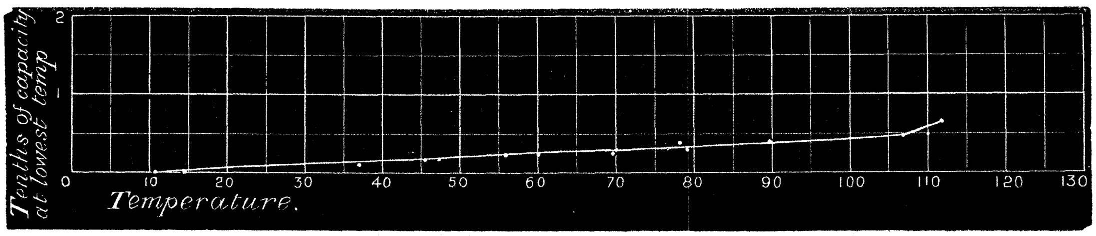
The insulation was constantly tested by an electrometer, and was found perfect throughout for temperatures below 110° C. Above this it began to give way.

The results are given in the following table and diagram (fig. 3).

Temperature.	Variable resistance.	Change of capacity.	Rate of increase per degree.
Fork making 99 complete vibrations per second.			
11° C.	4890		
37	4850	·0082	·00032
46	4830	·0122	·00035
56	4810	·0163	·00035
70	4790	·0204	·00035
79	4770	·0245	·00036
89	4750	·0286	·00035
107	4700	·0387	·00040
112	4600	·0591	·00059
Fork making 64 complete vibrations per second.			
14	5530		
48	5460	·0127	·00037
60	5440	·0163	·00035
70	5425	·0190	·00034
78	5390	·0253	·00039
89	5340	·0343	·00045
110	5270	·0470	·00049

The fact that both periods give the same value shows that, as was to be expected from its crystalline structure, there is no absorption in mica for those short times of charging.

Fig. 3.



Ebonite.

The ebonite condenser was made of twenty discs of ebonite, about a third of a millimetre thick, and three and a quarter inches in diameter. The ebonite sheets were carefully cleaned with paraffin dissolved in benzol, and this gave perfect insulation without any shellac border. At temperatures above 70° C. the capacity began to increase rapidly owing to softening. The highest temperature reached in the preliminary heating was a little under 80° C.

The insulation was constantly tested as in the case of mica, and was found perfect up to about 70° C.

The results are given in the following table and diagram (fig. 4).

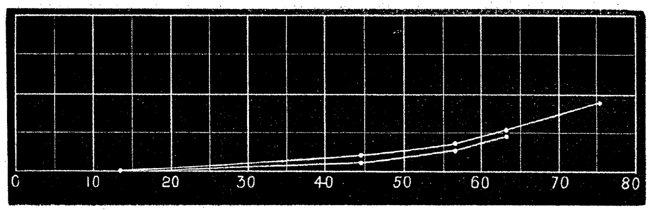
As the insulation was perfect, the greater capacity shown by the longer time of charging must be due to absorption—an effect which the structure of ebonite would lead us to expect. The column of values for instantaneous charge have been calculated on the assumption that the absorption goes on at the same rate from the beginning of the charge, as during the interval observed.

Temperature.	Variable resistance.	Change of capacity.	Rate of increase per degree.
Fork making 99 complete vibrations per second.			
13° C.	3520		
44	3460	·017	·00055
57	3415	·030	·00068
63	3355	·044	·00088
Fork making 64 complete vibrations per second.			
13	5250		
44	5143	·020	·00065
57	5060	·036	·00081
63	4960	·055	·00110
75	4800	·085	·00170

VALUES of rate of change corrected for instantaneous charge.

44° C.	·00037
57	·00043
63	·00048

Fig. 4.



Electrostatic Observations.—In the observations on ebonite by the electrostatic method the condenser consisted of two sheets of ebonite between the iron discs, and with a copper disc between them. The condenser rested upon a glass tripod inside the air bath, and the leading wires entered the air bath through glass tubes. The capacity of this was adjusted so as to be within the range of a fine sliding condenser in the Laboratory against which it was balanced; and the readings were taken on an Elliott electrometer. It required great care and patience to make so slow an electrometer suffice, for the reasons already stated. The insulation of all parts of the apparatus also required much attention.

The cold temperature, 8°, was obtained by running iced water round the bath containing the condenser. Subsequent observations showed that this had affected the insulation and so increased the apparent diminution of capacity.

The greater rate of change shown by the electrostatic results is probably due to the time of charging by the key worked by hand being greater than with the tuning fork.

Any leak would produce an apparent decrease of capacity by this method; so that, although the electrostatic results are of less weight than the others, the agreement of the results of two methods in which the main source of error tends in opposite directions is a confirmation of their accuracy.

The electrostatic results are given in the following table:—

Temperature.	Rate of change of capacity per degree.
17° C.	
40	·0008
48	·0010
50	·0016
57	·0015
(8	·0021)

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Glass.

The glass condenser was made with seventeen sheets of thin microscope-slide cover-glass, about three inches diameter. They consisted of a soda glass of high conductivity; so that, although the discs were carefully cleaned and bordered with shellac as in the case of mica, the insulatiou of the condenser, as tested by the electrometer, was never perfect. Since it is only the charging of the condenser that affects the galvanometer, the discharge passing sensibly all through D R (fig. 1), the defective insulation introduces simply a steady current during each time of charging. Each time the condenser is charged, the quantity of electricity passing D and B consists of (1) the charge of the condenser, and (2) the current through the condenser, which lasts during the time the vibrator P is in contact with S. For our present purpose, the current may be considered as immediately established at full strength when the circuit is closed. So that, neglecting absorption, the apparent capacity exceeds the true capacity by the conductivity of the condenser multiplied by the duration of the contact between P and S. And from observations on the apparent capacity with two forks of known speed, the true capacity can easily be found.

The highest temperature in the preliminary heating was less than 110° C.

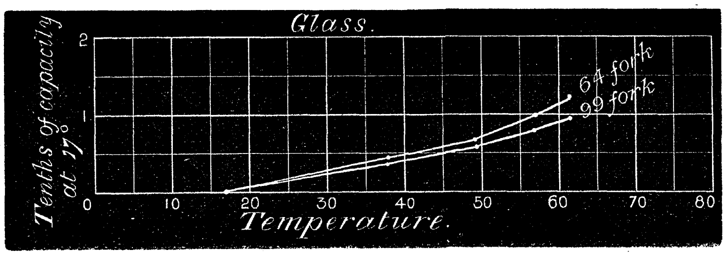
The results are given in the following table and diagram (fig. 5):—

Temperature.	Variable resistance.	Change of capacity.	Rate of change per degree.
Fork making 99 complete vibrations per second.			
17° C.	4795		
38	4630	·034	·0016
49	4540	·053	·0017
57	4425	·077	·0019
62	4320	·099	·0022
Fork making 64 complete vibrations per second.			
17	6070		
38	5830	·039	·0018
49	5670	·066	·0021
57	5470	·098	·0024
62	5360	·117	·0026

RATE of change corrected for instantaneous charge.

17° to 38°	·0013
49	·0010
57	·0010
62	·0015

Fig. 5.



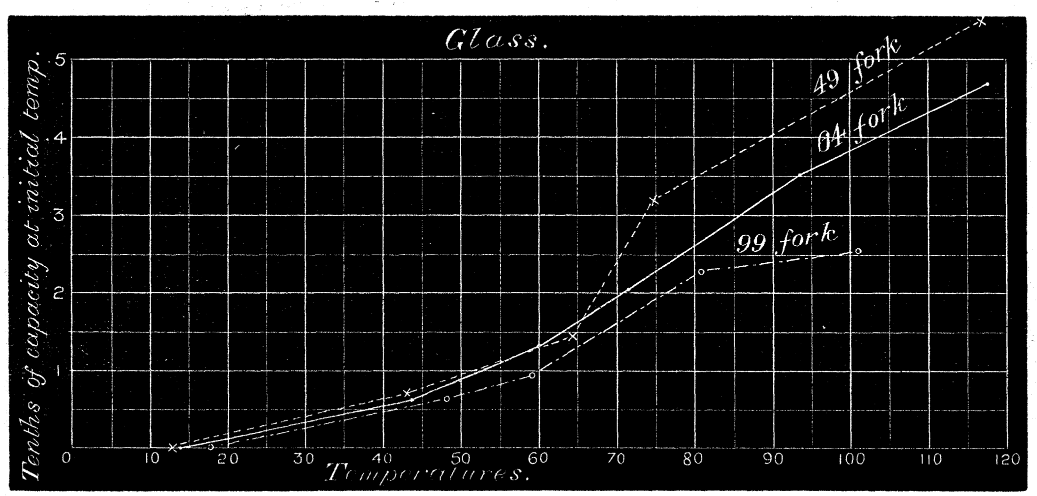
The next three tables and diagram give the results of experiments upon another condenser of microscope-slide cover-glass. This specimen of the glass had less conductivity than the previous, but the precautions for general insulation were not equal to those in the other experiments described. The condenser rested on a glass tripod inside the air bath, and the leading wires passed into the bath through glass tubes. The condenser consisted of twenty-two discs of glass cleaned and bordered with shellac in the usual way, with discs of lead foil between them.

The curves are almost exactly parallel up to between 40° and 50°, and diverge at higher temperatures in consequence of conduction; so that we may take the temperature change of specific inductive capacity for this specimen of glass to be about ·2 per cent. up to 50°.

An attempt was made to observe this temperature effect for shellac. The condenser was made by dipping the lead discs in shellac varnish, and carefully and thoroughly evaporating the alcohol. But after the condenser was made it was found that at a temperature below 50° the shellac softened, so that the plates were pressed together by the weights on the top of the condenser. To diminish the weight would have been useless; because, in any case, it would have been impossible to say how far a change of capacity was due to softening, and without a weight at all the results are quite unreliable.

Temperature.	Variable resistance.	Change of capacity in terms of capacity at lowest temperature.	Rate of change per degree.
With 99 fork.			
18° C.	4010		
48	3750	·0645	·00215
59	3630	·0941	·00230
81	3070	·2311	·00369
101	2890	·2541	·00251
With "64 fork" (between 64 and 65).			
14	6110		
44	5700	·0671	·00224
50	5560	·0900	·00250
60	5270	·1351	·00293
71·5	4850	·2062	·00362
94	3840	·3551	·00444
117	3230	·4712	·00506
With 49 fork.			
13	8100		
43	7480	·0765	·00255
64·5	7020	·1538	·00300
75	5850	·3201	·00516
115	4250	·5484	·00537

Fig. 6.

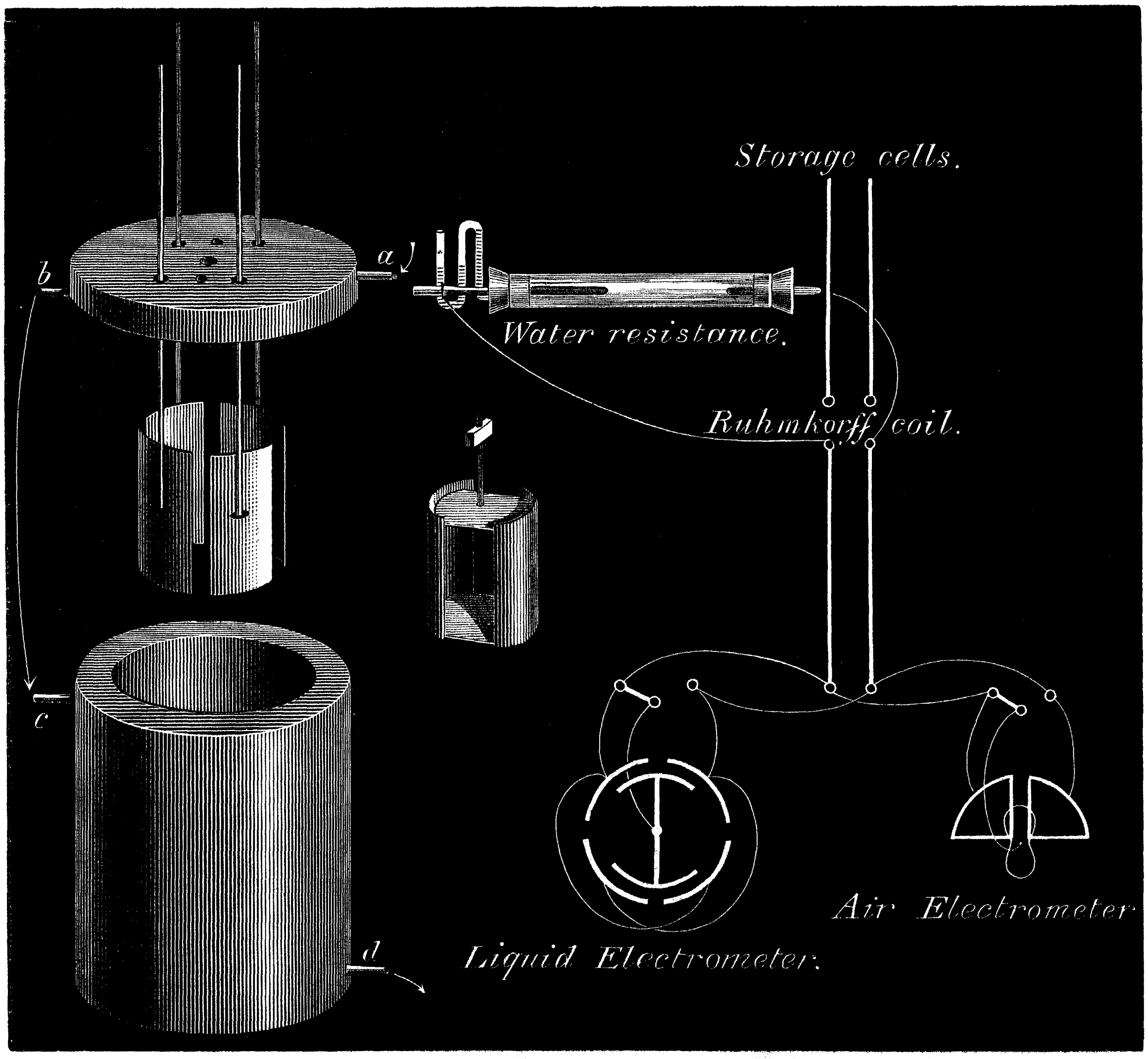


II. Liquids.

The apparatus for liquids consisted essentially of a quadrant electrometer immersed in the liquid.

The heating was done by a water bath; and on account of the highly inflammable character of some of the liquids experimented upon, the water was heated in a separate vessel at a distance, from the top and bottom of which pipes went to *a* and *d* in fig. 7. The hot water was pumped through in the direction *d c b a*.

Fig. 7.



The provision for the insulation was on a similar principle to that adopted in the case of solids. Each quadrant was supported by a long stiff rod, the upper part of which was fixed in a varnished glass tube. These four glass tubes were separately

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

clamped to a support about 18 inches above the surface of the liquid; so that the quadrants touched nothing except the liquid and this perfectly insulating support. The needle was suspended by a fine wire from the same height so as to oscillate under torsion.

The hollow top of the water bath had seven holes through it, as shown in the figure. Through the outside four passed, without touching, the rods supporting the quadrants, through the centre one hung the needle, and through the two next the centre thermometers could be inserted. This top was permanently fixed to the support of the needle and quadrants; and the lower part of the bath containing the liquid was moved up on a smooth sliding arrangement into contact with the top so as to immerse the needle and quadrants. Above the fixed top of the bath was a box with a window on one side, through which the movements of the needle and mirror were read by a scale and telescope. The deflections of the needle were observed when connected first to one pair of quadrants and then to the other pair.

As the needle and quadrants were parts of cylinders between three and four inches diameter and about a quarter of an inch apart, a large electromotive force was required to produce a deflection. The electromotive force had also to be rapidly reversed in direction to avoid as far as possible polarisation, convection, &c., in the liquid. The electromotive force was obtained from a Ruhmkorff coil without a condenser, and with a high resistance between the terminals to prevent sparking. This high resistance consisted of a wide glass tube, about 6 inches long, filled with distilled water, and having a thick copper wire sliding through a cork at each end. By altering the distance of the ends of the copper wires in the water the resistance could be adjusted and the deflection controlled as desired. The coil was worked by a current from the storage cells in the Laboratory.

As the electromotive force given by this arrangement was variable, and also the loss of electromotive force by conduction was different for each liquid and for each temperature, it was necessary to be independent of such changes. Accordingly a second electrometer was placed between the terminals just outside the liquid one, which being always in the same state gave the comparative values of the electromotive force. This second electrometer was an Elliott's lecture-room pattern with two quadrants removed and the needle connected to one of the remaining quadrants. The advantage of this form for the present purpose was that by moving the needle up from the quadrants the deflection could be diminished to any desired extent. Then the quotient of the deflection of the liquid electrometer by the deflection of the second electrometer was proportional to the specific inductive capacity of the liquid. For every deflection of the liquid electrometer a pair of deflections of the second electrometer were taken with the needle connected first to one pair of quadrants and then to the other. The general arrangement of the apparatus is shown in the figure.

The liquids experimented upon were turpentine, carbon bisulphide, glycerine, benzene, benzylene, olive oil, and paraffin oil. Methylated spirit was also tried; but its conductivity was so great that no deflection could be obtained. All except paraffin oil showed a decrease of specific inductive capacity with rise of temperature. The paraffin oil was that used in the lamps in the Laboratory, and its exceptional behaviour may have been due to some secondary action arising from impurity.

The results are given in the following tables and diagram. And to show the way in which the observations were made, the readings are given for turpentine at two temperatures.

	Temperature.	Ratio of specific inductive capacity to that at the lowest temperature.	Rate of decrease.
Turpentine:			
	20° C.	1·0000	
	36	·9800	·0012
	49	·9700	·0011
	62	·9600	·0009
Carbon bisulphide:			
	15	1·0000	
	35	·9130	·0040
	43	·8940	·0040
Glycerine:			
	18	1·0000	
	41	·8500	·0060
	61	·7760	·0053
Benzylene:			
	19	1·0000	
	41	·9870	·0006
	52	·9640	·0011
	63	·9350	·0015
Benzene:			
	15	1·0000	
	39	·9665	·0014
	58·5	·9507	·0012
Olive oil:			
	17	1·0000	
	38	·9530	·0021
	51	·9140	·0025
	59	·9130	·0021
	68	·8670	·0026

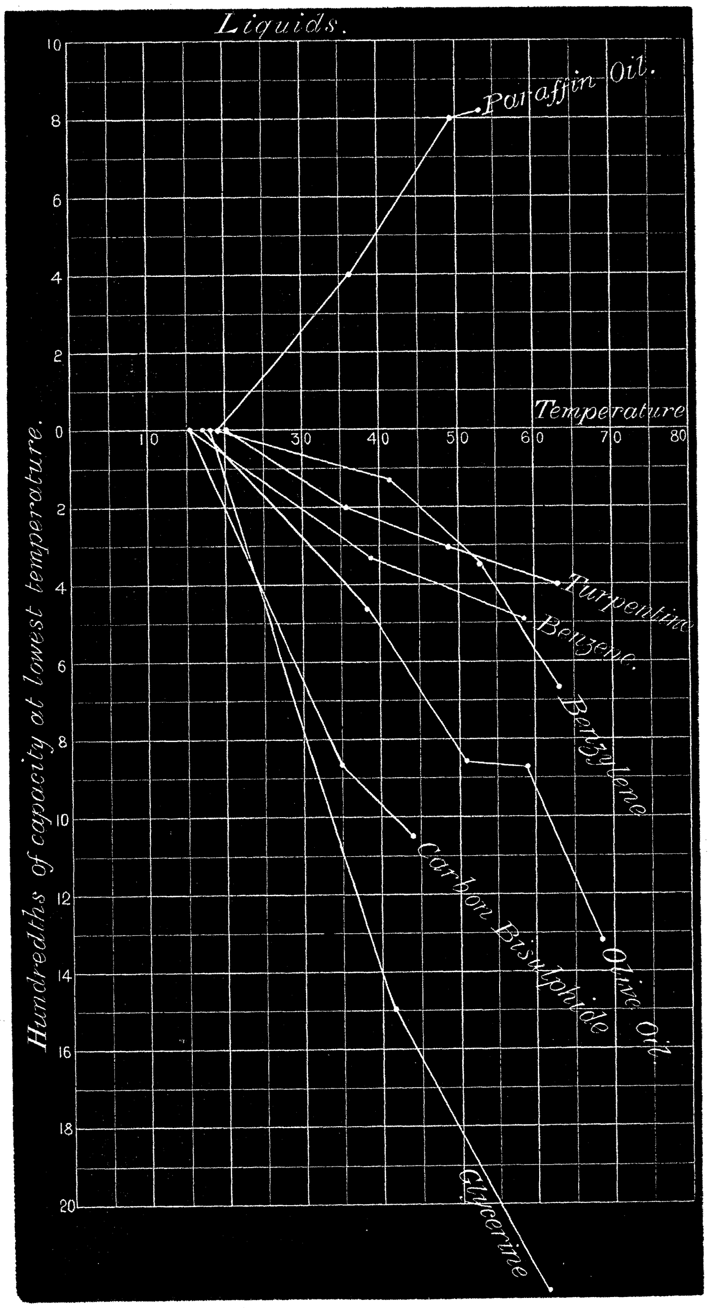
	Temperature.	Ratio of specific inductive capacity to that at the lowest temperature.	Rate of increase.
Paraffin oil:	18° C.	1·000	
	36·5	1·040	·0022
	49·5	1·080	·0025
	54	1·081	·0022

READINGS for Turpentine.

Temperature.	Liquid Electrometer.	Air electrometer.	Ratio of electrometer readings.
20°			
Zero	16·5		
Needle connected to one pair .	6·2	85	·098
,, ,, .	6·2	− 20	
,, other ,, .	26·6	− 21	·102
,, ,, .	26·0	75	
Zero	16·8		
			Mean ·100
36°			
Zero	17·3		
Needle connected to one pair .	26·3	83	·098
,, ,, .	26·3	− 15	
,, other ,, .	7·4	− 17	·098
,, ,, .	7·4	79	
			Mean ·098

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Fig. 8.



If the relation indicated by CLERK MAXWELL's electromagnetic theory of light held good, and the specific inductive capacity were equal to the square of the refractive index, then the rate of change of specific inductive capacity with temperature ought to be twice that of the refractive index. Amongst the liquids, for which DALE and GLADSTONE have observed the refractive indices at different temperatures, are four of those dealt with here. The mean rates of change, with temperature of the refractive index for the A line in the spectrum, deduced from their observations, are

Turpentine	·00035	for temperature range from	10° to 17°
Benzene	·00040	,, ,, ,,	10 ,, 39
Benzylene	·00037	,, ,, ,,	25 ,, 39
Glycerine	·00018	,, ,, ,,	20 ,, 48

Thus, it appears that although the two rates of change for Glycerine present no similarity whatever, those of the rest of the four are in a ratio not very far from 1 to 2, the approach being nearest in the case of Benzylene.

Appendix.

(Received October 18, 1889).

A suggestion having been made that, as the opposite effects of rise of temperature upon solids and liquids were observed by different methods, it would be well that both should be tested by the same method, a qualitative experiment was made on a solid by the method used for liquids, to see whether the result would agree with that already obtained.

A cylinder of glass was placed between the quadrants and needle of the liquid electrometer, leaving the needle free to oscillate; and observations were taken at different temperatures, exactly as already described for the case of a liquid dielectric. The result was always an increase of specific inductive capacity with rise of temperature. All the precautions for insulation, &c., were observed as in the experiments already described. But the heating was not maintained long enough to secure that the glass had acquired the full temperature of the air in the bath; so that the results obtained are only qualitative, the change being less than that corresponding to the temperature indicated by the air inside the bath. In one case, where the heating had lasted several hours, the rate of change rose as high as ·0024 per degree for a range of 50° C., a result sufficiently close to that obtained for glass by the other method.